

Natural Limits of Electroweak Model as Contraction of its Gauge Group

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Abstract

The low and higher energy limits of the Electroweak Model are obtained from first principles of gauge theory. Both limits are given by the same contraction of the gauge group, but for the different consistent rescalings of the field space. Mathematical contraction parameter in both cases is interpreted as energy. The very weak neutrino-matter interactions is explained by zero tending contraction parameter, which depend on neutrino energy. The second consistent rescaling corresponds to the higher energy limit of the Electroweak Model. At the infinite energy all particles lose masses, electroweak interactions become long-range and are mediated by the neutral currents. The limit model represents the development of the early Universe from the Big Bang up to the end of the first second.

1 Introduction

The modern theory of electroweak processes is the Electroweak Model, which is in good agreement with experimental dates, including the latest ones from LHC. This model is a gauge theory based on the gauge group $SU(2) \times U(1)$, which is the direct product of two simple groups. The operation of group contraction [1] transforms a simple or semisimple group to a nonsemisimple one. In particular the special unitary group $SU(2)$ is contracted to the group isomorphic to Euclid group $E(2)$ [2, 3]. For better understanding of a complicated physical system it is useful to investigate its behavior for limiting values of its physical parameters. In this paper we discuss mostly at the level of classical gauge fields the modified Electroweak Model with the contracted gauge group $SU(2; j) \times U(1)$. It was shown [4]–[7] that at low energies the contraction parameter depends on the energy s in center-of-mass system, so the contracted gauge group corresponds to the zero energy limit of the Electroweak Model. The very weak neutrinos-matter interactions and the linear dependence of their cross-section on neutrino energy both are explained from first principles of the Electroweak Model as contraction of its gauge group. But for the same contraction of the gauge group there is another consistent rescaling of the representation space, which lead to the infinite energy limit of the Electroweak Model. In this paper we consider both possibilities and discuss some particle properties in early Universe, where similar higher energies can exist.

2 Standard Electroweak Model

We shall follow the books [8]–[10] in description of standard Electroweak Model. Its Lagrangian is the sum of boson, lepton and quark Lagrangians

$$L = L_B + L_L + L_Q. \quad (1)$$

Boson sector $L_B = L_A + L_\phi$ involve two parts: the gauge field Lagrangian

$$L_A = \frac{1}{8g^2} \text{Tr}(F_{\mu\nu})^2 - \frac{1}{4}(B_{\mu\nu})^2 = -\frac{1}{4}[(F_{\mu\nu}^1)^2 + (F_{\mu\nu}^2)^2 + (F_{\mu\nu}^3)^2] - \frac{1}{4}(B_{\mu\nu})^2 \quad (2)$$

and the matter field Lagrangian

$$L_\phi = \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi - \frac{\lambda}{4}(\phi^\dagger\phi - v^2)^2, \quad (3)$$

where $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \in C_2$ are the matter fields. The covariant derivatives are given by

$$D_\mu\phi = \partial_\mu\phi - ig\left(\sum_{k=1}^3 T_k A_\mu^k\right)\phi - ig'Y B_\mu\phi, \quad (4)$$

where $T_k = \frac{1}{2}\tau_k$, $k = 1, 2, 3$ are generators of $SU(2)$, $Y = \frac{1}{2}\mathbf{1}$ is generator of $U(1)$, g and g' are constants. The gauge fields

$$A_\mu(x) = -ig\sum_{k=1}^3 T_k A_\mu^k(x), \quad B_\mu(x) = -ig'Y B_\mu(x) \quad (5)$$

take their values in Lie algebras $su(2)$, $u(1)$ respectively, and the stress tensors are as follows

$$F_{\mu\nu}(x) = \mathcal{F}_{\mu\nu}(x) + [A_\mu(x), A_\nu(x)], \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

To generate mass for the vector bosons the special mechanism of spontaneous symmetry breaking is used. One of L_B ground states

$$\phi^{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad A_\mu^k = B_\mu = 0$$

is taken as a vacuum state of the model, and small field excitations $v + \chi(x)$ with respect to this vacuum are regarded.

The fermion sector is represented by the lepton L_L and quark L_Q Lagrangians. The lepton Lagrangian is taken in the form

$$L_L = L_l^\dagger i\tilde{\tau}_\mu D_\mu L_l + e_r^\dagger i\tau_\mu D_\mu e_r - h_e[e_r^\dagger(\phi^\dagger L_l) + (L_l^\dagger \phi)e_r], \quad (6)$$

where $L_l = \begin{pmatrix} \nu_l \\ e_l \end{pmatrix}$ is the $SU(2)$ -doublet, e_r is the $SU(2)$ -singlet, h_e is constant, $\tau_0 = \tilde{\tau}_0 = \mathbf{1}$, $\tilde{\tau}_k = -\tau_k$, τ_μ are Pauli matrices and e_r, e_l, ν_l are two component Lorentz spinors. Last terms with factor h_e represent electron mass. The covariant derivatives are given by the formulas:

$$\begin{aligned} D_\mu L_l &= \partial_\mu L_l - i \frac{g}{\sqrt{2}} (W_\mu^+ T_+ + W_\mu^- T_-) L_l - \\ &- i \frac{g}{\cos \theta_w} Z_\mu (T_3 - Q \sin^2 \theta_w) L_l - ie A_\mu Q L_l, \\ D_\mu e_r &= \partial_\mu e_r - ig' Q A_\mu e_r \cos \theta_w + ig' Q Z_\mu e_r \sin \theta_w, \end{aligned} \quad (7)$$

where $T_\pm = T_1 \pm iT_2$, $Q = Y + T_3$ is the generator of electromagnetic subgroup $U(1)_{em}$, $Y = \frac{1}{2}\mathbf{1}$ is the hypercharge, $e = gg'(g^2 + g'^2)^{-\frac{1}{2}}$ is the electron charge and $\sin \theta_w = eg^{-1}$. The new gauge fields

$$\begin{aligned} Z_\mu &= \frac{1}{\sqrt{g^2 + g'^2}} (gA_\mu^3 - g'B_\mu), \quad A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g'A_\mu^3 + gB_\mu), \\ W_\mu^\pm &= \frac{1}{\sqrt{2}} (A_\mu^1 \mp iA_\mu^2) \end{aligned} \quad (8)$$

are introduced instead of (5).

The quark Lagrangian is given by

$$\begin{aligned} L_Q &= Q_l^\dagger i \tilde{\tau}_\mu D_\mu Q_l + u_r^\dagger i \tau_\mu D_\mu u_r + d_r^\dagger i \tau_\mu D_\mu d_r - \\ &- h_d [d_r^\dagger (\phi^\dagger Q_l) + (Q_l^\dagger \phi) d_r] - h_u [u_r^\dagger (\tilde{\phi}^\dagger Q_l) + (Q_l^\dagger \tilde{\phi}) u_r], \end{aligned} \quad (9)$$

where left quark fields form the $SU(2)$ -doublet $Q_l = \begin{pmatrix} u_l \\ d_l \end{pmatrix}$, right quark fields u_r, d_r are the $SU(2)$ -singlets, $\tilde{\phi}_i = \epsilon_{ik} \bar{\phi}_k$, $\epsilon_{00} = 1$, $\epsilon_{ii} = -1$ is the conjugate representation of $SU(2)$ group and h_u, h_d are constants. All fields u_l, d_l, u_r, d_r are two component Lorentz spinors. Last four terms with factors h_d and h_u specify the d - and u -quark mass. The covariant derivatives are given by

$$\begin{aligned} D_\mu Q_l &= \left(\partial_\mu - ig \sum_{k=1}^3 \frac{\tau_k}{2} A_\mu^k - ig' \frac{1}{6} B_\mu \right) Q_l, \\ D_\mu u_r &= \left(\partial_\mu - ig' \frac{2}{3} B_\mu \right) u_r, \quad D_\mu d_r = \left(\partial_\mu + ig' \frac{1}{3} B_\mu \right) d_r. \end{aligned} \quad (10)$$

From the viewpoint of electroweak interactions all known leptons and quarks are divided on three generations. Next two lepton generation are introduced in a similar way to (6). They are left $SU(2)$ -doublets

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_l, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_l, \quad Y = -\frac{1}{2} \quad (11)$$

and right $SU(2)$ -singlets: $\mu_r, \tau_r, Y = -1$. In addition to u - and d -quarks of the first generation there are (c, s) and (t, b) quarks of the next generations, which left fields

$$\begin{pmatrix} c_l \\ s_l \end{pmatrix}, \quad \begin{pmatrix} t_l \\ b_l \end{pmatrix}, \quad Y = \frac{1}{6}, \quad (12)$$

are described by the $SU(2)$ -doublets and the right fields are $SU(2)$ -singlets: $c_r, t_r, Y = \frac{2}{3}$; $s_r, b_r, Y = -\frac{1}{3}$. Their Lagrangians are introduced in a similar way to (9). Full lepton and quark Lagrangians are obtained by the summation over all generations. In what follows we shall regard only first generations of leptons and quarks.

3 Modified Electroweak Model

We consider a model where the contracted gauge group $SU(2; j) \times U(1)$ acts in the boson, lepton and quark sectors. The contracted group $SU(2; j)$ is obtained [11, 12] by the consistent rescaling of the fundamental representation of $SU(2)$ and the space C_2

$$z'(j) = \begin{pmatrix} jz'_1 \\ z'_2 \end{pmatrix} = \begin{pmatrix} \alpha & j\beta \\ -j\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} jz_1 \\ z_2 \end{pmatrix} = u(j)z(j),$$

$$\det u(j) = |\alpha|^2 + j^2|\beta|^2 = 1, \quad u(j)u^\dagger(j) = 1 \quad (13)$$

in such a way that the hermitian form

$$z^\dagger z(j) = j^2|z_1|^2 + |z_2|^2 \quad (14)$$

remains invariant, when contraction parameter tends to zero $j \rightarrow 0$ or is equal to the nilpotent unit $j = \iota$, $\iota^2 = 0$. The actions of the unitary group $U(1)$ and the electromagnetic subgroup $U(1)_{em}$ in the space $C_2(\iota)$ with the base $\{z_2\}$ and the fiber $\{z_1\}$ are given by the same matrices as on the space C_2 .

The space $C_2(j)$ of the fundamental representation of $SU(2; j)$ group can be obtained from C_2 by substituting z_1 by jz_1 . Substitution $z_1 \rightarrow jz_1$ induces another ones for Lie algebra generators $T_1 \rightarrow jT_1$, $T_2 \rightarrow jT_2$, $T_3 \rightarrow T_3$. As far as the gauge fields take their values in Lie algebra, we can substitute the gauge fields instead of transforming the generators, namely:

$$A_\mu^1 \rightarrow jA_\mu^1, \quad A_\mu^2 \rightarrow jA_\mu^2, \quad A_\mu^3 \rightarrow A_\mu^3, \quad B_\mu \rightarrow B_\mu. \quad (15)$$

Indeed, due to commutativity and associativity of multiplication by j

$$\begin{aligned} su(2; j) \ni \{A_\mu^1(jT_1) + A_\mu^2(jT_2) + A_\mu^3T_3\} = \\ = \{(jA_\mu^1)T_1 + (jA_\mu^2)T_2 + A_\mu^3T_3\}. \end{aligned} \quad (16)$$

For the gauge fields (8) these substitutions are as follows:

$$W_\mu^\pm \rightarrow jW_\mu^\pm, \quad Z_\mu \rightarrow Z_\mu, \quad A_\mu \rightarrow A_\mu. \quad (17)$$

The left lepton $L_l = \begin{pmatrix} \nu_l \\ e_l \end{pmatrix}$ and quark $Q_l = \begin{pmatrix} u_l \\ d_l \end{pmatrix}$ fields are $SU(2)$ -doublets, so their components are transformed in the similar way as components of the vector z , namely:

$$\nu_l \rightarrow j\nu_l, \quad e_l \rightarrow e_l, \quad u_l \rightarrow ju_l, \quad d_l \rightarrow d_l. \quad (18)$$

The right lepton and quark fields are $SU(2)$ -singlets and therefore are not changed.

After transformations (17), (18) and spontaneous symmetry breaking the boson Lagrangian (2),(3) can be represented in the form [11]–[13]

$$\begin{aligned} L_B(j) &= L_B^{(2)}(j) + L_B^{int}(j) = \\ &= \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}m_\chi^2 \chi^2 - \frac{1}{4}\mathcal{Z}_{\mu\nu}\mathcal{Z}_{\mu\nu} + \frac{1}{2}m_Z^2 Z_\mu Z_\mu - \\ &- \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu} + j^2 \left\{ -\frac{1}{2}\mathcal{W}_{\mu\nu}^+ \mathcal{W}_{\mu\nu}^- + m_W^2 W_\mu^+ W_\mu^- \right\} + \\ &+ L_B^{int}(j) = L_{B,b} + j^2 L_{B,f}, \end{aligned} \quad (19)$$

where as usual second order terms describe the boson particles content of the model and higher order terms $L_B^{int}(j)$ are regarded as their interactions. So Lagrangian (19) include charged W -bosons with identical mass $m_W = \frac{1}{2}gv$, massless photon A_μ , neutral Z -boson with the mass $m_Z = \frac{v}{2}\sqrt{g^2 + g'^2}$ and scalar Higgs boson χ , $m_\chi = \sqrt{2\lambda}v$.

In the limit $j \rightarrow 0$ Lagrangian (19) is split on two parts: Lagrangian of fields in the base

$$\begin{aligned} L_{B,b} &= \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}m_\chi^2 \chi^2 - \frac{1}{4}\mathcal{Z}_{\mu\nu}^2 + \frac{1}{2}m_Z^2 (Z_\mu)^2 - \\ &- \frac{1}{4}\mathcal{F}_{\mu\nu}^2 + \frac{gm_z}{2\cos\theta_W} (Z_\mu)^2 \chi - \lambda v \chi^3 + \\ &+ \frac{g^2}{8\cos^2\theta_W} (Z_\mu)^2 \chi^2 - \frac{\lambda}{4}\chi^4 \end{aligned} \quad (20)$$

and Lagrangian of fields in the fiber

$$\begin{aligned} L_{B,f} &= -\frac{1}{2}\mathcal{W}_{\mu\nu}^+ \mathcal{W}_{\mu\nu}^- + m_W^2 W_\mu^+ W_\mu^- - \\ &- 2ig \left(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+ \right) (\mathcal{F}_{\mu\nu} \sin\theta_W + \\ &+ \mathcal{Z}_{\mu\nu} \cos\theta_W) - \frac{i}{2}e \left[A_\mu \left(\mathcal{W}_{\mu\nu}^+ W_\nu^- - \mathcal{W}_{\mu\nu}^- W_\nu^+ \right) - \right. \\ &- A_\nu \left(\mathcal{W}_{\mu\nu}^+ W_\mu^- - \mathcal{W}_{\mu\nu}^- W_\mu^+ \right) \left. \right] + g W_\mu^+ W_\mu^- \chi - \\ &- \frac{i}{2}g \cos\theta_W \left[Z_\mu \left(\mathcal{W}_{\mu\nu}^+ W_\nu^- - \mathcal{W}_{\mu\nu}^- W_\nu^+ \right) - \right. \\ &- Z_\nu \left(\mathcal{W}_{\mu\nu}^+ W_\mu^- - \mathcal{W}_{\mu\nu}^- W_\mu^+ \right) \left. \right] + \end{aligned}$$

$$\begin{aligned}
& +\frac{g^2}{4} \left(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+ \right)^2 + \frac{g^2}{4} W_\mu^+ W_\nu^- \chi^2 - \\
& -\frac{e^2}{4} \left\{ \left[\left(W_\mu^+ \right)^2 + \left(W_\mu^- \right)^2 \right] (A_\nu)^2 - \right. \\
& -2 \left(W_\mu^+ W_\nu^+ + W_\mu^- W_\nu^- \right) A_\mu A_\nu + \\
& + \left[\left(W_\nu^+ \right)^2 + \left(W_\nu^- \right)^2 \right] (A_\mu)^2 \left. \right\} - \\
& -\frac{g^2}{4} \cos \theta_W \left\{ \left[\left(W_\mu^+ \right)^2 + \left(W_\mu^- \right)^2 \right] (Z_\nu)^2 - \right. \\
& -2 \left(W_\mu^+ W_\nu^+ + W_\mu^- W_\nu^- \right) Z_\mu Z_\nu + \\
& + \left[\left(W_\nu^+ \right)^2 + \left(W_\nu^- \right)^2 \right] (Z_\mu)^2 \left. \right\} - \\
& -eg \cos \theta_W \left\{ W_\mu^+ W_\mu^- A_\nu Z_\nu + W_\nu^+ W_\nu^- A_\mu Z_\mu - \right. \\
& -\frac{1}{2} \left(W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^- \right) (A_\mu Z_\nu + A_\nu Z_\mu) \left. \right\}. \tag{21}
\end{aligned}$$

The lepton Lagrangian (6) in terms of electron and neutrino fields takes the form

$$\begin{aligned}
L_L(j) = & e_l^\dagger i \tilde{\tau}_\mu \partial_\mu e_l + e_r^\dagger i \tau_\mu \partial_\mu e_r - m_e (e_r^\dagger e_l + e_l^\dagger e_r) + \\
& + \frac{g \cos 2\theta_w}{2 \cos \theta_w} e_l^\dagger \tilde{\tau}_\mu Z_\mu e_l - e e_l^\dagger \tilde{\tau}_\mu A_\mu e_l - \\
& - g' \cos \theta_w e_r^\dagger \tau_\mu A_\mu e_r + g' \sin \theta_w e_r^\dagger \tau_\mu Z_\mu e_r + \\
& + j^2 \left\{ \nu_l^\dagger i \tilde{\tau}_\mu \partial_\mu \nu_l + \frac{g}{2 \cos \theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \right. \\
& + \frac{g}{\sqrt{2}} \left[\nu_l^\dagger \tilde{\tau}_\mu W_\mu^+ e_l + e_l^\dagger \tilde{\tau}_\mu W_\mu^- \nu_l \right] \left. \right\} = L_{L,b} + j^2 L_{L,f}. \tag{22}
\end{aligned}$$

The quark Lagrangian (9) in terms of u - and d -quarks fields can be written as

$$\begin{aligned}
L_Q(j) = & d_l^\dagger i \tilde{\tau}_\mu \partial_\mu d_l + d_r^\dagger i \tau_\mu \partial_\mu d_r - m_d (d_r^\dagger d_l + d_l^\dagger d_r) - \\
& - \frac{e}{3} d_l^\dagger \tilde{\tau}_\mu A_\mu d_l - \frac{g}{\cos \theta_w} \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) d_l^\dagger \tilde{\tau}_\mu Z_\mu d_l - \\
& - \frac{1}{3} g' \cos \theta_w d_r^\dagger \tau_\mu A_\mu d_r + \frac{1}{3} g' \sin \theta_w d_r^\dagger \tau_\mu Z_\mu d_r - \\
& + j^2 \left\{ u_l^\dagger i \tilde{\tau}_\mu \partial_\mu u_l + u_r^\dagger i \tau_\mu \partial_\mu u_r - m_u (u_r^\dagger u_l + u_l^\dagger u_r) + \right. \\
& + \frac{g}{\cos \theta_w} \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u_l^\dagger \tilde{\tau}_\mu Z_\mu u_l +
\end{aligned}$$

$$\begin{aligned}
& + \frac{2e}{3} u_l^\dagger \tilde{\tau}_\mu A_\mu u_l + \frac{g}{\sqrt{2}} \left[u_l^\dagger \tilde{\tau}_\mu W_\mu^+ d_l + d_l^\dagger \tilde{\tau}_\mu W_\mu^- u_l \right] + \\
& + \frac{2}{3} g' \cos \theta_w u_r^\dagger \tau_\mu A_\mu u_r - \frac{2}{3} g' \sin \theta_w u_r^\dagger \tau_\mu Z_\mu u_r \Big\} = \\
& = L_{Q,b} + j^2 L_{Q,f},
\end{aligned} \tag{23}$$

where $m_e = h_e v / \sqrt{2}$ and $m_u = h_u v / \sqrt{2}$, $m_d = h_d v / \sqrt{2}$ represent electron and quark masses.

The full Lagrangian of the modified model is given by the sum

$$\begin{aligned}
L(j) &= L_B(j) + L_Q(j) + L_L(j) = \\
&= L_{B,b} + L_{L,b} + L_{Q,b} + j^2 \{L_{B,f} + L_{L,f} + L_{Q,f}\} = \\
&= L_b + j^2 L_f,
\end{aligned} \tag{24}$$

where L_b is Lagrangian of the base fields and L_f is Lagrangian of the fiber fields.

The boson Lagrangian $L_B(j)$ was discussed in [11, 12] on the level of classical fields, where it was shown that masses of all particles involved in the Electroweak Model remain the same under contraction $j^2 \rightarrow 0$. In this limit the contribution $j^2 L_f$ of neutrino, W -boson and u -quark fields as well as their interactions with other fields to the Lagrangian (24) will be vanishingly small in comparison with contribution L_b of electron, d -quark and remaining boson fields. So Lagrangian (24) describes very weak interaction of neutrino fields with the matter. On the other hand, contribution of the neutrino part $j^2 L_f$ to the full Lagrangian is risen when the parameter j^2 is increased, that again corresponds to the experimental facts. So contraction parameter can be phenomenologically connected with neutrino energy.

4 Decompositions of physical systems and group contractions

The standard way of describing a physical system in field theory is its decomposition on independent more or less simple subsystems and then introduction of interactions between them. In Lagrangian formalism this imply that some terms describe independent subsystems (free fields) and the rest terms correspond to interactions between fields. When subsystems are not interacted with each other the composed system is a formal unification of subsystems and symmetry group of the whole system is direct product $G = G_1 \times G_2$, where G_1 and G_2 are symmetry groups of the subsystems. The Electroweak Model gives an example of such approach. Indeed, there are free boson, lepton, quark fields in Lagrangian and terms which describe interactions between these fields.

The operation of group contraction transforms a simple or semisimple group G to a nonsemisimple one with the structure of a semidirect product $G = A(\times G_1)$, where A is Abelian and $G_1 \subset G$ is untouched subgroup. At the same time the

space of fundamental representation of group G is split under contraction in such a way that subgroup G_1 acts in the fiber. Gauge theory with contracted gauge group describe a physical system, which break up into two subsystems S_b and S_f . One subsystem S_b include all fields from the base and other subsystem S_f is built from fiber fields. S_b form the close system since according to semi-Riemannian geometry [14, 15] properties of the base do not depend on points of the fiber, what means on the physical language that fields from the fiber do not interact with fields from the base. But on the contrary properties of the fiber depend on points of the base, therefore the subsystem S_b exert influence upon S_f . More precisely fields from the base are outer (or background) fields for subsystem S_f and specify outer conditions in every fiber.

In particular, the simple group $SU(2)$ is contracted to the nonsemisimple one $SU(2; \iota)$, which is isomorphic to the Euclid group $E(2) = A_2(\times SO(1))$, where Abelian subgroup A_2 is generated by translations [11, 12]. The fields space of the standard Electroweak Model is split after the contraction in such a way that neutrino, W -boson and u -quark fields are in the fiber, whereas all other fields are in the base.

In order to avoid terminological misunderstanding let us stress that we regard locally trivial splitting, which is defined by the projection in the field space. This splitting is understood in the context of semi-Riemannian geometry [14, 15], where properties of the base do not depend on the fiber and has nothing to do with the principal fiber bundle.

The simple and best known example of fiber space is the nonrelativistic space-time with one dimensional base, which is interpreted as time, and three dimensional fiber, which is interpreted as proper space. It is well known, that in nonrelativistic physics the time is absolute and does not depend on the space coordinates, while the space properties can be changed in time. The simplest demonstration of this fact is Galilei transformation $t' = t$, $x' = x + vt$. The space-time of the special relativity is transformed to the nonrelativistic space-time when dimensionfull parameter — velocity of light c — tends to the infinity and dimensionless parameter $\frac{v}{c}$ tends to zero.

5 Weak neutrino-matter interactions and the physical interpretation of the parameter j

To discover the connection of gauge group contraction with limiting case of the Electroweak Model and establish the physical meaning of the contraction parameter we need more fine consideration on the level of quantized fields. Namely, we discuss neutrino elastic scattering on electron and quarks. The corresponding diagrams for the neutral and charged currents interactions are represented in Fig. 1 and Fig. 2.

Under substitutions (17),(18) both vertex of diagram in Fig. 1, a) are multiplied by j^2 , as it follows from lepton Lagrangian (22). The propagator of virtual fields W according to boson Lagrangian (19) is multiplied by j^{-2} . Indeed, propagator is

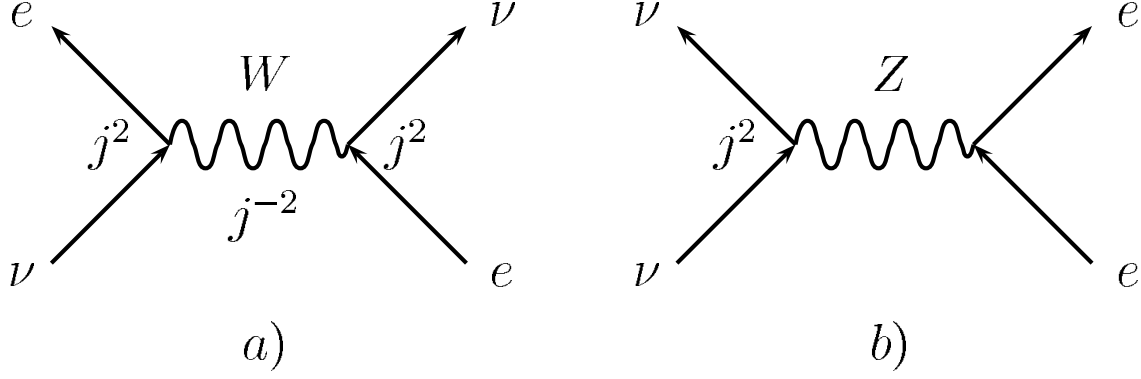


Figure 1: Neutrino elastic scattering on electron

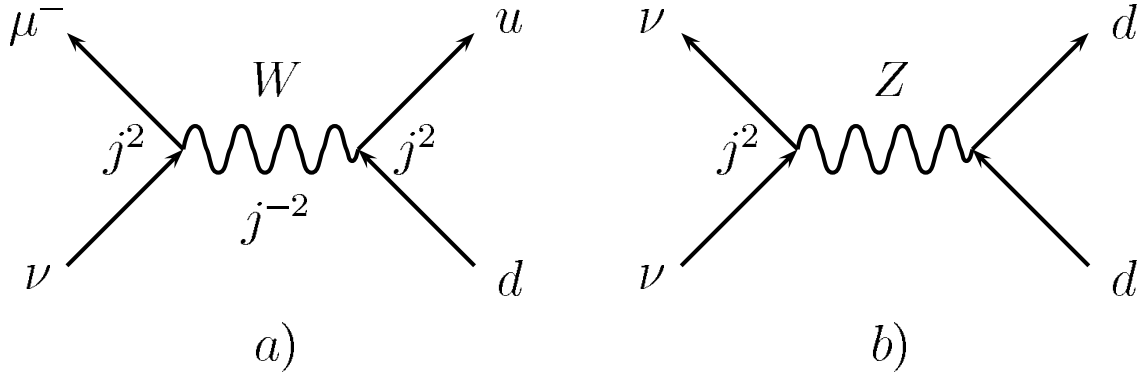


Figure 2: Neutrino elastic scattering on quarks

inverse operator to operator of free field, but the later for W -fields is multiplied by j^2 .

So in total the probability amplitude for charged weak current interactions is transformed as $\mathcal{M}_W \rightarrow j^2 \mathcal{M}_W$. For diagram in Fig. 1, b) only one vertex is multiplied by j^2 , whereas second vertex and propagator of Z virtual field do not changed, so the corresponding amplitude for neutral weak current interactions is transformed in a similar way $\mathcal{M}_Z \rightarrow j^2 \mathcal{M}_Z$. A cross-section is proportionate to an squared amplitude, so neutrino-electron scattering cross-section is proportionate to j^4 . For low energies $s \ll m_W^2$ this cross-section is as follows [9]

$$\sigma_{\nu e} = G_F^2 s f(\xi) = \frac{g^4}{m_W^4} \tilde{f}(\xi), \quad (25)$$

where $G_F = 10^{-5} \frac{1}{m_p^2} = 1,17 \cdot 10^{-5} \text{ GeV}^{-2}$ is Fermi constant, s is squared energy in central mass system, $\xi = \sin \theta_W$, $\tilde{f}(\xi) = f(\xi)/32$ is function of Weinberg angle. The cross-section in the laboratory system for neutrino energy $m_e \ll E_\nu \ll m_W$ is given by [16]

$$\sigma_{\nu e} = G_F^2 m_e E_\nu \tilde{g}(\xi). \quad (26)$$

On the other hand, taking into account that contraction parameter is dimensionless, we can write down

$$\sigma_{\nu e} = j^4 \sigma_0 = (G_F s)(G_F f(\xi)) \quad (27)$$

and obtain

$$j^2(s) = \sqrt{G_F s} \approx \frac{g\sqrt{s}}{m_W}. \quad (28)$$

Neutrino elastic scattering on quarks due to neutral and charged currents are pictured in Fig. 2. Cross-sections for neutrino-quarks scattering are obtained in a similar way as for the lepton case and are as follows [9]

$$\sigma_\nu^W = G_F^2 s \hat{f}(\xi), \quad \sigma_\nu^Z = G_F^2 s h(\xi). \quad (29)$$

Nucleons are some composite construction of quarks, therefore some form-factors are appeared in the expressions for neutrino-nucleons scattering cross-sections. The final expression

$$\sigma_{\nu n} = G_F^2 s \hat{F}(\xi) \quad (30)$$

coincide with (25), i.e. this cross-section is transformed as (27) with the contraction parameter (28). At low energies scattering interactions make the leading contribution to the total neutrino-matter cross-section, therefore it has the same properties (27),(28) with respect to contraction of the gauge group. So, the very weak neutrinos-matter interactions and the linear dependence of their cross-section on neutrino energy both are explained by the Electroweak Model with the contracted gauge group.

6 High-Energy Lagrangian of Electroweak Model

We shown in previous sections that contraction $j \rightarrow 0$ of the gauge group (13) of the Electroweak Model corresponds to its zero energy limit. In this limit the first components of the lepton and quark doublets become infinite small in comparison with their second components. On the contrary, when energy increase the first components of the doublets become greater then their second ones. In the infinite energy limit the second components of the lepton and quark doublets will be infinite small as compared with their first components. To describe this limit we introduce new contraction parameter ϵ and **new consistent rescaling** of the group $SU(2)$ and the space C_2 as follows

$$z'(\epsilon) = \begin{pmatrix} z'_1 \\ \epsilon z'_2 \end{pmatrix} = \begin{pmatrix} \alpha & \epsilon\beta \\ -\epsilon\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} z_1 \\ \epsilon z_2 \end{pmatrix} = u(\epsilon)z(\epsilon),$$

$$\det u(\epsilon) = |\alpha|^2 + \epsilon^2|\beta|^2 = 1, \quad u(\epsilon)u^\dagger(\epsilon) = 1 \quad (31)$$

Both contracted groups $SU(2; j)$ (13) and $SU(2; \epsilon)$ (31) are the same and are isomorphic to Euclid group $E(2)$, but the space $C_2(\epsilon)$ is split in the limit $\epsilon \rightarrow 0$ on

the one-dimension base $\{z_1\}$ and the one-dimension fiber $\{z_2\}$. From the mathematical point of view it is not important first or second Cartesian axis forms the base of fibering and in this sence constructions (13) and (31) are equivalent. But the doublet components are interpreted as a certain physical fields, therefore the fundamental representations (13) and (31) of the same contracted unitary group lead to the different limit cases of the Electroweak Model, namely, its zero energy and infinite energy limits.

In the second contraction scheme (31) all gauge bosons are transformed according to the rules (17) with the natural substitution of j by ϵ . Instead of (18) the lepton and quark fields are transformed now as follows

$$e_l \rightarrow \epsilon e_l, \quad d_l \rightarrow \epsilon d_l, \quad \nu_l \rightarrow \nu_l, \quad u_l \rightarrow u_l. \quad (32)$$

The next reason for inequality of the first and second doublet components is the special mechanism of spontaneous symmetry breaking, which is used to generate mass of vector bosons and other elementary particles of the model. In this mechanism one of Lagrangian ground states $\phi^{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix}$ is taken as vacuum of the model and then small field excitations $v + \chi(x)$ with respect to this vacuum are regarded. So Higgs boson field χ and constant v are multiplied by ϵ . As far as masses of all particles are proportionate to v we obtain the following transformation rule for the contraction (31)

$$\chi \rightarrow \epsilon \chi, \quad v \rightarrow \epsilon v, \quad m_p \rightarrow \epsilon m_p, \quad p = \chi, W, Z, e, u, d. \quad (33)$$

After transformations (17), (32)–(33) the boson Lagrangian of the Electroweak Model can be represented in the form

$$\begin{aligned} L_B(\epsilon) = & -\frac{1}{4}\mathcal{Z}_{\mu\nu}^2 - \frac{1}{4}\mathcal{F}_{\mu\nu}^2 + \epsilon^2 L_{B,2} + \epsilon^3 g W_\mu^+ W_\mu^- \chi + \epsilon^4 L_{B,4}, \\ L_{B,4} = & m_W^2 W_\mu^+ W_\mu^- - \frac{1}{2}m_\chi^2 \chi^2 - \lambda v \chi^3 - \frac{\lambda}{4}\chi^4 + \\ & + \frac{g^2}{4} (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+)^2 + \frac{g^2}{4} W_\mu^+ W_\nu^- \chi^2, \\ L_{B,2} = & \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}m_Z^2 (Z_\mu)^2 - \frac{1}{2}\mathcal{W}_{\mu\nu}^+ \mathcal{W}_{\mu\nu}^- + \\ & + \frac{gm_z}{2\cos\theta_W} (Z_\mu)^2 \chi + \frac{g^2}{8\cos^2\theta_W} (Z_\mu)^2 \chi^2 - \\ & - 2ig (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) (\mathcal{F}_{\mu\nu} \sin\theta_W + \mathcal{Z}_{\mu\nu} \cos\theta_W) - \\ & - \frac{i}{2}e \left[A_\mu (\mathcal{W}_{\mu\nu}^+ W_\nu^- - \mathcal{W}_{\mu\nu}^- W_\nu^+) + \frac{i}{2}e A_\nu (\mathcal{W}_{\mu\nu}^+ W_\mu^- - \mathcal{W}_{\mu\nu}^- W_\mu^+) \right] - \\ & - \frac{i}{2}g \cos\theta_W \left[Z_\mu (\mathcal{W}_{\mu\nu}^+ W_\nu^- - \mathcal{W}_{\mu\nu}^- W_\nu^+) - Z_\nu (\mathcal{W}_{\mu\nu}^+ W_\mu^- - \mathcal{W}_{\mu\nu}^- W_\mu^+) \right] - \end{aligned}$$

$$\begin{aligned}
& -\frac{e^2}{4} \left\{ \left[(W_\mu^+)^2 + (W_\mu^-)^2 \right] (A_\nu)^2 - \right. \\
& -2 (W_\mu^+ W_\nu^+ + W_\mu^- W_\nu^-) A_\mu A_\nu + \left[(W_\nu^+)^2 + (W_\nu^-)^2 \right] (A_\mu)^2 \Big\} - \\
& -\frac{g^2}{4} \cos \theta_W \left\{ \left[(W_\mu^+)^2 + (W_\mu^-)^2 \right] (Z_\nu)^2 - \right. \\
& -2 (W_\mu^+ W_\nu^+ + W_\mu^- W_\nu^-) Z_\mu Z_\nu + \left[(W_\nu^+)^2 + (W_\nu^-)^2 \right] (Z_\mu)^2 \Big\} - \\
& -eg \cos \theta_W \left[W_\mu^+ W_\mu^- A_\nu Z_\nu + W_\nu^+ W_\nu^- A_\mu Z_\mu - \right. \\
& \left. -\frac{1}{2} (W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-) (A_\mu Z_\nu + A_\nu Z_\mu) \right]. \tag{34}
\end{aligned}$$

The lepton Lagrangian in terms of electron and neutrino fields takes the form

$$\begin{aligned}
L_L(\epsilon) &= L_{L,0} + \epsilon^2 L_{L,2} = \\
&= \nu_l^\dagger i \tilde{\tau}_\mu \partial_\mu \nu_l + e_r^\dagger i \tau_\mu \partial_\mu e_r + g' \sin \theta_w e_r^\dagger \tau_\mu Z_\mu e_r - \\
&- g' \cos \theta_w e_r^\dagger \tau_\mu A_\mu e_r + \frac{g}{2 \cos \theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \\
&+ \epsilon^2 \left\{ e_l^\dagger i \tilde{\tau}_\mu \partial_\mu e_l - m_e (e_r^\dagger e_l + e_l^\dagger e_r) + \right. \\
&+ \frac{g \cos 2\theta_w}{2 \cos \theta_w} e_l^\dagger \tilde{\tau}_\mu Z_\mu e_l - e e_l^\dagger \tilde{\tau}_\mu A_\mu e_l + \\
&\left. + \frac{g}{\sqrt{2}} (\nu_l^\dagger \tilde{\tau}_\mu W_\mu^+ e_l + e_l^\dagger \tilde{\tau}_\mu W_\mu^- \nu_l) \right\}. \tag{35}
\end{aligned}$$

The quark Lagrangian in terms of u- and d-quarks fields can be written as

$$\begin{aligned}
L_Q(\epsilon) &= L_{Q,0} - \epsilon m_u (u_r^\dagger u_l + u_l^\dagger u_r) + \epsilon^2 L_{Q,2}, \\
L_{Q,0} &= d_r^\dagger i \tau_\mu \partial_\mu d_r + u_l^\dagger i \tilde{\tau}_\mu \partial_\mu u_l + u_r^\dagger i \tau_\mu \partial_\mu u_r - \\
&- \frac{1}{3} g' \cos \theta_w d_r^\dagger \tau_\mu A_\mu d_r + \frac{1}{3} g' \sin \theta_w d_r^\dagger \tau_\mu Z_\mu d_r + \\
&+ \frac{2e}{3} u_l^\dagger \tilde{\tau}_\mu A_\mu u_l + \frac{g}{\cos \theta_w} \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u_l^\dagger \tilde{\tau}_\mu Z_\mu u_l + \\
&+ \frac{2}{3} g' \cos \theta_w u_r^\dagger \tau_\mu A_\mu u_r - \frac{2}{3} g' \sin \theta_w u_r^\dagger \tau_\mu Z_\mu u_r, \\
L_{Q,2} &= d_l^\dagger i \tilde{\tau}_\mu \partial_\mu d_l - m_d (d_r^\dagger d_l + d_l^\dagger d_r) - \\
&- \frac{e}{3} d_l^\dagger \tilde{\tau}_\mu A_\mu d_l - \frac{g}{\cos \theta_w} \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) d_l^\dagger \tilde{\tau}_\mu Z_\mu d_l +
\end{aligned}$$

$$+ \frac{g}{\sqrt{2}} \left[u_l^\dagger \tilde{\tau}_\mu W_\mu^+ d_l + d_l^\dagger \tilde{\tau}_\mu W_\mu^- u_l \right]. \quad (36)$$

The complete Lagrangian of the modified model is given by the sum $L(\epsilon) = L_B(\epsilon) + L_L(\epsilon) + L_Q(\epsilon)$ and can be written in the form

$$L(\epsilon) = L_\infty + \epsilon L_1 + \epsilon^2 L_2 + \epsilon^3 L_3 + \epsilon^4 L_4. \quad (37)$$

Unlike zero energy limit (24) the Electroweak Model demonstrate for $\epsilon \rightarrow 0$ five stages of behavior, which are distinguished by powers of the contraction parameter. In the infinite energy limit ($\epsilon = 0$) Lagrangian is equal to

$$\begin{aligned} L_\infty = & -\frac{1}{4} \mathcal{Z}_{\mu\nu}^2 - \frac{1}{4} \mathcal{F}_{\mu\nu}^2 + \nu_l^\dagger i \tilde{\tau}_\mu \partial_\mu \nu_l + u_l^\dagger i \tilde{\tau}_\mu \partial_\mu u_l + \\ & + e_r^\dagger i \tau_\mu \partial_\mu e_r + d_r^\dagger i \tau_\mu \partial_\mu d_r + u_r^\dagger i \tau_\mu \partial_\mu u_r + L_\infty^{int}(A_\mu, Z_\mu), \end{aligned} \quad (38)$$

where

$$\begin{aligned} L_\infty^{int}(A_\mu, Z_\mu) = & \frac{g}{2 \cos \theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \frac{2e}{3} u_l^\dagger \tilde{\tau}_\mu A_\mu u_l + \\ & + \frac{g}{\cos \theta_w} \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u_l^\dagger \tilde{\tau}_\mu Z_\mu u_l + \\ & + g' \sin \theta_w e_r^\dagger \tau_\mu Z_\mu e_r - g' \cos \theta_w e_r^\dagger \tau_\mu A_\mu e_r - \\ & - \frac{1}{3} g' \cos \theta_w d_r^\dagger \tau_\mu A_\mu d_r + \frac{1}{3} g' \sin \theta_w d_r^\dagger \tau_\mu Z_\mu d_r + \\ & + \frac{2}{3} g' \cos \theta_w u_r^\dagger \tau_\mu A_\mu u_r - \frac{2}{3} g' \sin \theta_w u_r^\dagger \tau_\mu Z_\mu u_r. \end{aligned} \quad (39)$$

The limit model includes only **massless particles**: photons A_μ and neutral bosons Z_μ , left quarks u_l and neutrinos ν_l , right electrons e_r and quarks u_r, d_r . The electroweak interactions become long-range because are mediated by the massless neutral Z -bosons and photons.

The infinite energies can exist only in the initial moment of creation when the Universe is point-like [17, 18], as depicted in figure 3, and it is not clear what means long-range electroweak interactions. However more interesting is the Universe evolution and the limit Lagrangian L_∞ can be considered as a good approximation near the Big Bang just as the nonrelativistic mechanics is a good approximation of the relativistic one at low velocities. From the explicit form of the interaction part $L_\infty^{int}(A_\mu, Z_\mu)$ it follows that there are no interactions between particles of different kind, for example neutrinos interact only with each other by neutral currents. It looks like some stratification of the Electroweak Model with only one sort of particles in each stratum.

It is well known that to gain a better understanding of a physical system it is usefull to investigate its properties for limiting values of physical parameters. It follows from (37) that there are five stages in formation of the Electroweak Model after the creation of the Universe which are distinguished by the powers of the

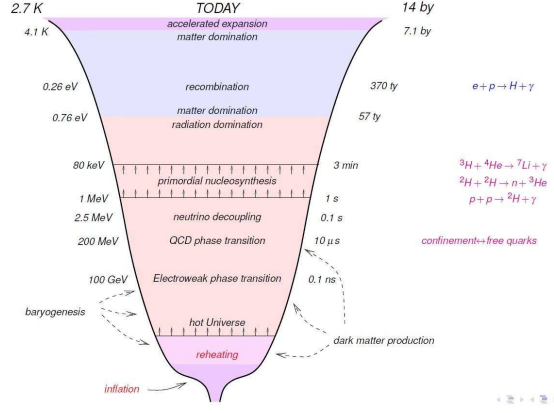


Figure 3: History of the Universe ($1\text{eV} = 10^4 K$) [19]

contraction parameter ϵ . This offers an opportunity for construction of intermediate limit models. One can take the Lagrangian L_∞ for the initial limit system, then add $L_1 = -m_u(u_r^\dagger u_l + u_l^\dagger u_r)$ and obtain the second limit model with the Lagrangian $\mathcal{L}_1 = L_\infty + \epsilon L_1$. After that one can add L_2 and obtain the third limit model $\mathcal{L}_2 = L_\infty + \epsilon L_1 + \epsilon^2 L_2$. The last limit model has the Lagrangian $\mathcal{L}_3 = L_\infty + \epsilon L_1 + \epsilon^2 L_2 + \epsilon^3 L_3$. But it should be noted that among all limit models only L_∞ is gauge model with the gauge group isomorphic to Euclid group $E(2)$.

Already at the level of classical gauge fields we can conclude that the u -quark first restores its mass in the evolution of the Universe. Indeed the mass term of u -quark in the Lagrangian (37) is proportional to the first power ϵL_1 , whereas the mass terms of Z -boson, electron and d -quark are multiplied by the second power of the contraction parameter

$$\epsilon^2 \left[\frac{1}{2} m_Z^2 (Z_\mu)^2 + m_e (e_r^\dagger e_l + e_l^\dagger e_r) + m_d (d_r^\dagger d_l + d_l^\dagger d_r) \right]. \quad (40)$$

Higgs boson and charged W -boson, whose mass terms are multiplied by ϵ^4 , restore their masses after all other particles of the Electroweak Model.

7 Conclusion

We have investigated the low and higher energy limits of the Electroweak Model which are obtained from first principles of gauge theory as contraction of its gauge group. Above limits are given by the same contraction of the gauge group, but for the different consistent rescalings of the representation space. It was shown that mathematical contraction parameter in both cases is interpreted as typical energy.

The very weak neutrino-matter interactions especially at low energies can be explained by this model already at the level of classical (non-quantum) gauge fields.

The zero tending contraction parameter is connected with neutrino energy and reproduce the linear energy dependence of the neutrino-matter cross-section.

The alternative rescaling of the gauge group and the field space corresponds to the infinite energy limit of the Electroweak Model, which goes in this limit through the five stages depending on the powers of the contraction parameters. At the infinite energy all particles are massless and electroweak interactions become long-range. But the infinite energies can exist only in the Big Bang, i.e. in the initial moment of creation when the Universe is point-like and it is not clear what means long-range. However more interesting is the Universe development and the limit Lagrangian L_∞ can be considered as a good approximation near the Big Bang just as the nonrelativistic mechanics is a good approximation of the relativistic one at low velocities. Particularly we can conclude that according to the Electroweak Model u -quark first restores its mass among other particles in the evolution of the Universe.

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